MLE for Receiver-to-Receiver Time Synchronization in Wireless Networks with Exponential Distributed Delays

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***Abstract*—Receiver-to-receiver time synchronization in wireless networks is considered, and appropriate maximum-likelihood estimators (MLE) for environments with exponential distrusted reception delays are proposed. In the receiver-to-receiver synchronization approach, time at receivers should be directly related to one another without referring to the sender (reference), which permits to eliminate the sender's uncertainty from the variable delays (time critical-path). The models and estimators proposed for the sender-to-receiver approach are thus inappropriate for the receiver-to-receiver one. A model that accurately reflects the relative feature of the considered approach and eliminates the senders's uncertainty is used, where timestamps at the receivers are directly related without referring to the sender's time or timestamps. By directly relating time at two receivers with identical exponential reception delay, $Exp(\lambda)$, it yields a $Laplace(0, 1/\lambda)$ distribution as the difference between the two delays. They use the log likelihood function of the latter and the ML method, the offset estimator has been analytically derived, where a linear program is given for the joint offset/skew model. The accuracy of the proposed estimators has been numerically analyzed by simulation. Results show high precision of the proposed estimators, which can be integrated with any receiver-to-receiver synchronization protocol.**

I. Introduction and Background

The receiver-to-receiver approach for time synchronization exploits the broadcast property of the wireless medium [1], [2], where receivers within the vicinity of the same sender receive a broadcast message at approximately the same time, with very little variability due to the reception timstamping jitter at the reception. The principle is to use a sequence of synchronization signals (beacons) that are periodically broadcasted from a given and fixed sender- termed reference- and intercepted by synchronizing nodes (receivers). The nodes timestamp the arrival-time of each beacon using their local clocks, then every pair of nodes exchange the recorded timestamps to construct samples for estimation. Fig. 1 illustrates the construction of one samples in the two approaches. In the sender-to-receiver, the transmitter (node n_1) timestamps the first packet as t_1, the reception of this packet at n_2 is timestamped by the latter, t_2, and then the transmission/reception of the reply packet are respectively timestamped t_3, t_4. The tuple (t_1, t_2, t_3, t_4) is used as a sample for estimation. In the receiver-to-receiver approach, a reference is used to broacast messages and only reception timestamps are used to construct samples (t'_1, t'_2).

As show in Fig 1, the receiver-to-receiver approach has the advantage of reducing the time-critical path, and therefore improving synchronization accuracy compared to the sender-to-receiver approach [3, P. 294]. The time-critical path is the latency that contributes to non-deterministic errors when exchanging synchronization signals. The time-critical path in case of the first approach is the result of the following four factors that can vary non-deterministically [4]: i) Send time, which is the time spent by the sender for message construction added to the time spent to transmit the message from the sender's host to the network interface. ii) Access time; the time spent for medium access at the MAC layer. iii) Propagation time, as the time for the message to reach the receiver once it has left the sender's radio. iv) Receive time, which is the time spent by the receiver to process the message. The receiver-to-receiver approach removes the send time and the access time from the critical path [3, P. 294], which represent the two largest sources of non-determinism. This reduces the effect of delay variation.

A variety of estimators has been proposed for the sender-to-receiver approach, such as [5], [6], [7], [8], etc., and even for multi-carrier environments [9], but few has been done for the receiver-to-receiver approach. The reader may refer to [10] for a detailed review on existing estimators. Most works dealing with the receiver-to-receiver synchronization are empirical and use simplified models. Exponential and Gaussian distribution for reception delays are the most common distributions used in the literature. Gaussian distribution is appropriate for environment with no queuing latency, where the exponential one is suitable when the reception involves queuing [8]. Estimators for Gaussian delays in wireless sensor networks have been considered in our previous work [11], generalize for multi-hop environments and implemented in [12]. Yigitler et al [13] considered the same distortion but for environments with correlated delays. Environments with heterogeneous nodes have been considered in [14], where estimators for general mesh networks have been proposed. However, all these estimators do not apply for environments with exponential delays that results in different likelihood functions.

To our knowledge, Sari et al. [15] are the first and the only ones who considered maximum-likelihood estimation for receiver-to-receiver synchronization with exponential delays. The major problem we observed on their model is the fact

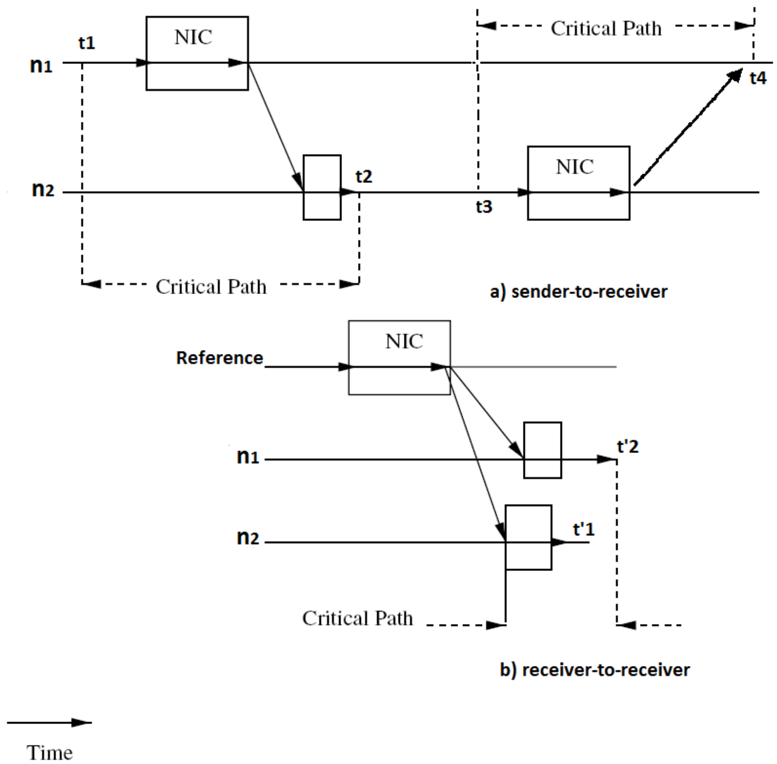

Fig. 1: Receiver-to-receiver vs. sender-to-receiver

that it does not synchronize receivers directly but through synchronization to the reference clock, which completely deviates from the receiver-to-receiver concept. Refer to equations in [15, p. 1] that relate the i^{th} timestamps at two receivers, say X and Y, to that of the transmitter (reference),

$$X[i] = T_1 + \theta_x + \beta_x \tau[i] + v_{x,\lambda_x}[i], \tag{1}$$

$$Y[i] = T_1 + \theta_y + \beta_y \tau[i] + v_{y,\lambda_y}[i], \tag{2}$$

where T_1 stands for the time on the transmitter when it sends the first synchronization signal, θ_x and β_x, respectively θ_y and β_y, stand for the offset and skew between the clocks of the receiver node, X, respectively, node Y, and the transmitter (reference). $\tau[i]$ stands for the difference between T_1 and the time of the i^{th} synchronization signal transmission, with respect to the transmitter's clock. This way, receivers' timestamps are related to the sender's clock at the moment of transmitting the th signal. Consequently, the exponential distributed random variables v_{x,λ_x}, v_{y,λ_y} would be the noise (variable delay) due to *both transmission and reception*. This adds transmission delays to the time-critical-path and completely deviates from the receiver-to-receiver concept, where the concept of a reference is for signal broadcasting, and not for a common reference of time. Eliminating the sender's uncertainty from the critical path is vital in receiver-to-receiver synchronization, which cannot be achieved by relating the transmitter's (reference's) time to the receiver's time in the model.

The model presented hereafter– which is based on Eq. (3) and Eq. (4)– allows to directly estimating relative parameters without using or referring to the reference time. This faithfully reflects the receiver-to-receiver synchronization by eliminating the sender's latency from the model's random variables (rv). Contrary to Sari et al. model, the parameter related to the delay distribution (λ) is supposed unknown in the remaining of the paper. Sari et al. suppose this parameter is known, which is unrealistic. They claim that when it is unknown, the estimators would be similar to those of [5], which is inaccurate; First, [5] is limited to the offset-only model, where Sari et al. have been dealing with the joint skew/offset model. Second, it is proposed for the sender-to-receiver and needs two-way time exchange (between the sender and the receiver) to eliminate the assumed symmetric delays. It is not possible to construct such samples with the receiver-to-receiver approach[1].

II. Proposed Estimators

Considering the synchronization between two nodes, n_1, and n_2, and suppose another node, n_3, that acts as a reference for broadcasting beacons. The problem consists in using a set of K samples of reception timestamps, say (u_i, v_i), $i \in \{1, ..., K\}$, to estimate the relative skew and/or offset (depending on the model) between nodes, n_1, and n_2. In the offset-only model, the timestamps are related by [4],

$$u_i = v_i + \theta + d_{ui} - d_{vi}, \tag{3}$$

[1] referring to Fig. 1, it is not possible tuples (t_1, t_2, t_3, t_4), with the receiver-to-receiver principle

where θ denotes the relative offset between the two nodes, d_{ui} and d_{vi} denote the reception delays for the i^{th} samples respectively at nodes, n_1 and n_2, u_i and v_i denote the corresponding timestamps.

The joint model involves two parameters; the relative skew, say α ($\alpha > 0$), and the relative offset, say β. Timestamps (u_i, v_i) are related by [4],

$$u_i = \alpha v_i + \beta + d_{ui} - d_{vi}. \tag{4}$$

As in [5], it is supposed that each of d_{ui} (respectively d_{vi}) is composed of a fixed portion, say fd_{ui} (respectively fd_{vi}), and a variable portion, say X_{ui} (respectively X_{vi}). It is very important to note here that these delays are merely related to the receivers (reception delays), as the sender's time is not involved in the model, contrary to that of [15]. This explains the elimination of the sender's uncertainty in the proposed model.

The fixed portions are assumed to be equal and the variable ones to be exponential distributed random variables (rv) with unknown mean, i.e., $X_{vi}, X_{ui} \sim Exp(\lambda)$. It follows that $d_{ui} - d_{vi} = X_{ui} - X_{vi}$. Let this difference be denoted by X_i. As the latter is the difference between two Exponential rv with parameter, λ, it follows a Laplace distribution, $Laplace(0, 1/\lambda)$ [16]. In the following, estimators for the two models are derived.

A. *Offset-Only Model*

Eq. (3) yields,

$$X_i = u_i - v_i - \theta, \tag{5}$$

where θ denotes the relative offset between the two nodes. The likelihood function of K Laplace distributed samples, $\mathcal{L}(\theta|X_1, ...X_K)$, is given by,

$$\mathcal{L}(\theta|X_1, ...X_K) = \prod_{i=1}^{K} \frac{1}{2/\lambda} e^{\frac{-|X_i|}{1/\lambda}}$$

$$\mathcal{L}(\theta|X_1, ...X_K) = \prod_{i=1}^{K} \frac{\lambda}{2} e^{-\lambda|u_i - v_i - \theta|}. \tag{6}$$

The log-likelihood is then given by,

$$\ln(\mathcal{L}(\theta|X_1, ...X_K)) = n\ln(\lambda/2) - \lambda \sum_{i=1}^{K} |u_i - v_i - \theta|. \tag{7}$$

Maximizing Eq. (7) is equivalent to minimize $\sum_{i=1}^{K} |u_i - v_i - \theta|$.

Let $u_i - v_i$ be denoted by w_i, and let us order the new samples, $w_i, i \in 1, .., k$. Following the canonical order, the new samples are again denoted by $w^i, i \in 1, .., k$ i.e. $w^1 \leq w^2 \leq ...w^K \leq$.

Minimizing the previous expression is equivalent to minimizing, $\sum_{i=1}^{K} |\theta - w^i|$.

The minimum of such function has been given in [17], which is,

$$\begin{cases} i) w^{(K+1)/2}, & if \quad K \quad is \quad odd, \\ ii) [w^{(K/2)}, w^{K/2+1}] & if \quad K \quad is \quad even. \end{cases} \tag{8}$$

Generally speaking, this is equivalent to the interval: $[\lfloor w^{(K+1)/2} \rfloor, \lceil w^{(K+1)/2} \rceil]$, which reduces to a single number when K is odd.

It results,

$$\begin{aligned} \widehat{\theta}_{mle} &= \arg\max(\ln \mathcal{L}(\theta|X_1, ...X_K)) \\ &= [\lfloor w^{(K+1)/2} \rfloor, \lceil w^{(K+1)/2} \rceil]. \end{aligned} \tag{9}$$

In practice, it is possible to always use an odd number for the sample size to ensure having a single optimum as an estimator.

B. *Joint Offset-Skew Model*

Eq. (4) yields,

$$X_i = u_i - \alpha v_i - \beta. \tag{10}$$

The likelihood function for K samples, $\mathcal{L}(\alpha, \beta|X_1, ...X_K)$, becomes,

$$\mathcal{L}(\alpha, \beta|X_1, ...X_K) = \prod_{i=1}^{K} \frac{\lambda}{2} e^{-\lambda|u_i - \alpha v_i - \beta|}. \tag{11}$$

It results,

$$\ln(\mathcal{L}(\alpha, \beta|X_1, ...X_K)) = n\ln(\lambda/2) - \lambda \sum_{i=1}^{K} |u_i - \alpha v_i - \beta|. \tag{12}$$

Maximizing Eq. (12) is equivalent to minimize, $\sum_{i=1}^{K} |u_i - \alpha v_i - \beta|$.

This is similar to minimizing the least absolute deviation problem, which is nonlinear but can be transformed to a linear program (LP) by adding K variables, $y_i, i \in \{1, .., K\}$, which permits to remove the absolute values [18]. K other variables, x_i, are then added to transform inequalities into equalities, along with the variable β as the difference of two nonnegative new variables, i.e. $\beta = \beta_1 - \beta_2, \beta_1, \beta_2 \geq 0$. The final standard LP is given by,

$$\begin{cases} min \sum_{i=1}^{K} y_i. \\ y_i - x_i + \alpha v_i - \beta_1 + \beta_2 = u_i, \forall i \in \{1, .., K\} \\ y_i - t_i - \alpha v_i + \beta_1 - \beta_2 = -u_i, \forall i \in \{1, .., K\} \\ y_i, x_i, t_i \geq 0, \forall i \in \{1, .., K\}, \alpha > 0, \beta_1, \beta_2 \geq 0. \end{cases} \tag{13}$$

To obtain estimators, $\widehat{\alpha}_{mle}$, $\widehat{\beta}_{mle}$, the LP given by Eq. (13) can be resolved using any iterative method (e.g. Simplex, Karmakar, etc.) [18], once numerical values of the samples (u_i, v_i) are fixed,

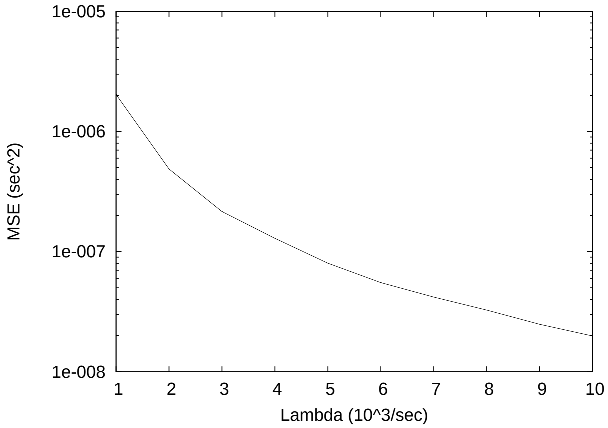

Fig. 2: MSE of θ_{mle} vs. λ, $\theta = 0.1sec$

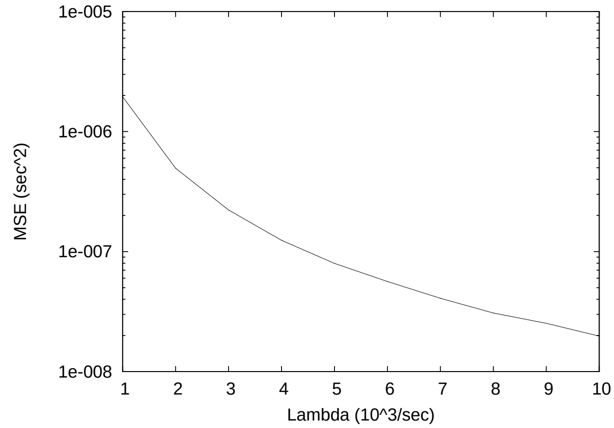

Fig. 3: MSE of θ_{mle} vs. λ, $\theta = 1sec$

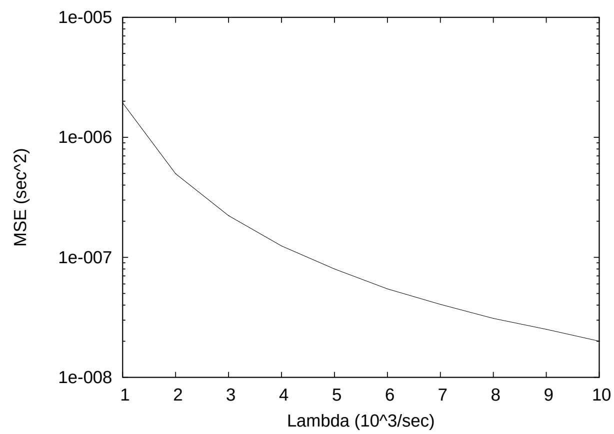

Fig. 4: MSE of θ_{mle} vs. λ, $\theta = 10sec$

III. Numerical Analysis

The mean square error (MSE) of the proposed estimator in the offset-only model is numerically analyzed in this section by simulation using MATLAB. The number of samples that are used for estimation has been fixed to 10, which was sufficient enough to get precise estimation (higher values didn't provide any significant additional precision in the simulated scenarios). Real offset (θ) between two nodes has been fixed to $0.1sec$, $1sec$, and $10sec$, and the corresponding MSE of the estimated offset, θ_{mle}, has been plotted respectively in Fig. 2, Fig. 3, Fig. 4. Exponential rv with parameter λ ($Exp(\lambda)$) have been generated in every execution. Note that this parameter is the reverse of the mean delay, μ, (i.e., $\mu = 1/\lambda$). Each point of the plots is the average of 10^4 executions with different random sequences for the same value of λ, and the resulted error bars have been in a much lower scale than the MSE values and are invisible even with 99% confidence interval. The results confirm high precision of the estimator that normally increases (i.e., the error decreases) with the increase of λ. The MSE starts at the order of 10^{-6} for low values of λ, i.e. μ at the order of $1ms$, and it reaches as low values as the order of 10^{-8} for high values of λ. The three plots have the same shape and the errors are at the same order, which demonstrates that the precision is unaffected by the real offset to be estimated. The joint skew/offset model, results in the same precision. The only difference is that in the offset-only model continuous re-estimation of the parameters is required when clocks drift, while the joint model captures clock drifting and allows to avoid frequent re-synchronization.

IV. Conclusion

Maximum likelihood estimators for receiver-to-receiver time synchronization in wireless networks with exponential distributed delays has been considered in this paper. It has been shown that estimators such as those proposed in [15] deviate from the receiver-to-receiver synchronization concept and fail to eliminate the sender's delay, as they relate the receiver timestamps to the sender (reference). They also suppose values of parameters related to delay distribution to be known a priory. Motivated by this shortcomings, new estimators have been proposed in this paper. Contrary to the model in [15], the one used herein directly relates the timestamps of the synchronizing nodes, i.e. without using the reference clock. This faithfully eliminates the sender uncertainty from the time-critical path and thus reduces the jitters, which helps reducing the estimators' errors. Both the offset-only and the joint skew/offset models have been considered, and the value of the parameter related to the random delay distribution is not necessary known, but analytically eliminated by the MLE method. An analytical expression for the offset estimator in the offset-only model has been provided, while a standard linear program has been derived for the joint model, which enables calculation of the estimators using any iterative algorithm once values of the parameters (samples) are fixed. The estimator provided for the offset-only model has the simplicity advantage, which facilitates its calculation, notably in constrained networks, e.g. wireless sensor networks. However, this model does not take into account the clock drift, and it is thus instable and requires frequent update of the estimator. The joint model captures the clock drift and permits to derive more stable estimators that would be reliable for much longer period before an update is needed [3]. Nonetheless, the estimators in our case are to be obtained after resolving a linear programming, which is costly for resource constrained networks. The use of the model in such networks should be limited to a short window on the sample size instead of relying on a high sample size. An interesting perspective is to propose simpler estimators for such networks. Real application of the estimators to a synchronization protocol is also a potential perspective.

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