Estimators for RBS-Based Time Synchronization in Heterogeneous Wireless Networks

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***Abstract*—A general scenario for using reference broadcast synchronization (RBS) in heterogeneous multi-hop wireless networks is defined, together with an appropriate model allowing to directly derive maximum likelihood estimators (MLE) to the skew and offset between communicating nodes. The model captures nodes heterogeneity that may cause differences in the reception delays. It also eliminates the need for synchronizing to the reference and enables direct relative synchronization. This accurately eliminates sender's delays from the critical path, which faithfully reflects RBS features and the receiver/receiver paradigm. It will be shown that state-of-the-art RBS estimators for the joint skew/offset problem fail to eliminate sender's delay. Appropriate MLEs are accordingly proposed. The Cramer-Rao lower bounds (CRLBs) are calculated, then the proposed estimators are numerically analyzed. Results show the estimators quadratically converge to the CRLB as the sample size rises, and that the precession is uninfluenced by the degree of heterogeneity (increase of delay differences). In a summary, the contribution of this paper is threefold; i) considering usage of RBS in heterogeneous and multi-hop networks, ii) defining an appropriate model and- accordingly- the maximum likelihood estimators for relative skew/offset between communicating nodes, iii) defining theoretical bounds for the estimators and numerically analyzing their convergence.**

I. Introduction

Time synchronization in wireless networks is a challenging research topic [1]. Many state-of-the-art distributed protocols are based on two-way exchange of synchronization signals. This approach is known as sender/receiver synchronization [2]. On the other hand, the Reference Broadcast Synchronization (RBS) protocol [3] is one of the milestone solutions proposed thus far, and it is conceptually different from the previous approach. It introduced the receiver/receiver synchronization, which relies on the broadcast property of the wireless communication medium. With this property, receivers located within listening distance to the same sender (reference) would physically capture a broadcast signal at approximately the same time. Still, timestamping may witness some variability due to the reception delays at the receivers. RBS uses a sequence of synchronization signals (beacons) from a fixed reference, which allows potential receivers to mutually estimate relative offset and skew of their respective clocks. The reference periodically broadcasts signals that are received at the synchronizing nodes, and timestamped with local clocks. The local timestamps are exchanged between the nodes and used to construct samples for estimating relative skews/offsets.

The protocol reduces time-critical path, which is the path of a message that contributes to non-deterministic errors in a protocol [4]. With sender/receiver protocols, the time involved in sending a message (signal) from a sender to a receiver is the result of the following four factors that can vary non-deterministically: i) Send time, which is the time spent by the sender for message construction along with the time spent to transmit the message from the sender's host to the network interface. ii) Access time; the time spent for medium access at the MAC layer. iii) Propagation time, as the time necessary for the message to reach the receiver once it has left the sender. This time is relatively negligible. iv) Receive time, which is the time spent by the receiver to process the message. RBS removes the send time and the access time from the critical path; the two largest sources of non-determinism. This would contribute to a high degree of synchronization, provided estimators are well-established. First and foremost, and as eliminating delays related to the sender is the essence of RBS, any model upon which the estimators are derived must eliminate such delays. It is inevitable to use relative synchronization, instead of synchronization to the reference. It will be detailed later how synchronizing receivers to the reference (which is performed by some state-of-the-art models) violates RBS property. The model proposed in this paper considers this problem.

Although RBS has been proposed for wireless sensor networks, it can be generalized for use in any wireless network with a broadcast medium. In this paper, a general scenario of wireless networks where RBS can be used is first defined. The scenario considers possible heterogeneous nodes. With heterogeneous nodes, it is inappropriate to assume reception delays are identical. In other words, assuming that delay differences (regarding reference signal reception) between two nodes converge to zero. A model that takes this constraint into account is defined. The model is used to derive the maximum likelihood estimators (MLE) of skew/offset, which enables any couple of potential receivers to mutually and relatively synchronize their timestamping. The corresponding Cramer-Rao lower bounds (CRLB) are derived and numerically compared to the estimators' mean square errors (MSE). Results demonstrate fast convergence to the optimum (CRLB).

The remainder of this paper is organized as follows. Section II presents the related work, followed by a network and synchronization model for the use of RBS-based synchronization in multi-hop wireless networks. Section IV proposes MLE for the synchronization parameters (skew/offset). In Section V the CRLB is calculated and a numerical analysis is provided, which investigates the convergence of the proposed estimators to the CRLB (the optimum). Finally, Section VI draws the conclusions and enumerates the potential perspectives.

II. Related work

Time synchronization is vital for many applications in communication networks and distributed systems, where there is no common reference of time (clock). In fact, each entity runs its own clock that inevitably drifts off through the time. Attempting to synchronize local clocks to a universal time (absolute time) is extremely difficult and impractical [5]. It is more rational to keep clocks running independently and provide mechanisms to allow each entity to estimate the time at another entity given its own reading. If necessary, an accurate clock reflecting the universal time may be one of these (other) entities. All synchronization protocols rely on timestamp exchange, which inevitably yields delays. The key challenge that faces all these protocols is delay variability and unpredictability. Communication delays is more variable and unpredictable in wireless environments, making it more challenging to ensure fine-grained synchronization and accurately estimate relevant parameters. We focus on estimators in this sections; readers interested in a review on synchronization protocols of wireless networks can refer to [2].

Most of estimators proposed thus far are devoted to sender/receiver synchronization, where the two synchronizing entities (nodes) perform a two-way message exchange of timestamps. Some estimators are based on the offset-only model [6], which needs to exchange synchronization signals and to update offsets at a high frequency. This is costly and impractical in wireless environments. Noh et al. [7] provide general offset/skew maximum likelihood estimators (MLE), where only the first and the last observations of timing message exchanges are used. The approach was generalized in [8]. The authors suppose delays at all nodes to follow a Gaussian distribution with the same mean. A similar mathematical approach is followed in this paper, to derive estimators of RBS protocol. However, the model used herein, and consequently all the derived estimators and bounds, are completely different. The model of [7] does not apply to RBS synchronization but only to sender/receiver based solutions. Moreover, the estimators in this paper consider delay difference between nodes, where [7] supposes reception delays at nodes to have the same mean. RAT [9] is a sender/receiver synchronization protocol, which has been integrated with B-MAC to incrementally improve its performance. The authors used linear regression to estimate synchronization parameters. Since the authors' aim is to reduce the preamble period at the MAC layer, the estimators provide a weak synchronization.

RBS [3], as described in Section I, defines a completely different approach- receiver/receiver- that reduces the message critical path. The authors empirically observed delays to be Gaussian and consequently estimate the offset by the average, which is well-known to be the MLE for the Gaussian distribution. Since then, few has been done for RBS parameter estimation, notably for the joint skew/offset model. To our knowledge, the work of Sari et al. [10] is the only one that considers joint skew/offset MLE estimators for RBS. It assumes exponentially distributed delays. The major shortcomings one can notice on this model is that it does not synchronize receivers directly but through synchronization to the reference's clock, which completely deviates from the RBS concept. Note that the concept of a reference in RBS is only to have a common reference for signal broadcast, but not a common reference of time. The authors claim that after synchronizing the receivers to the reference, the relative parameters can be determined. This is possible but with cumulative errors on the estimators; and more importantly, each estimator would not eliminate sender's delays. Eliminating the sender uncertainty from the critical path is at the core of RBS, while this uncertainty is not eliminated by relating the transmitter (reference) and the receiver times in the model [1]. The model used herein is different from the one of Sari et al. It allows to directly estimating relative parameters without using or refering to the reference clock. This faithfully reflects RBS concept.

III. Network and synchronization Model

The network model is presented in this section. Without loss of generality, a wireless mesh network is considered. The proposed RBS-extension, as well as the estimators proposed in next section, apply to any wireless network with the node-heterogeneous feature. Only the wireless segment of the mesh network is considered. The network architecture is depicted on Figure 1. Multiple nodes are supposed to act as reference broadcasters to cover the network area. A simplified projection would be a single-hop WLAN with a single reference. In the general mesh network configuration, the reference can be a base-station, an access point, a sink, or even a dedicated hardware for synchronization signal broadcasting. The solution makes abstraction of this entity and applies to all configurations. Within each reference's coverage area, there are several ordinary nodes and mesh routers. The ordinary nodes may range from laptops, to PDA and cell-phones, to sensor motes. This makes them completely heterogeneous in terms of computation and bandwidth. Therefore, the expected reception delay of synchronization signals would vary from a node to another. Let denote the coverage area of a reference, say R, by Cov_R. A node located within two or more references' coverage areas is termed a potential synchronization gateway. A gateway can be either a mesh router or a regular node, as long as it acts as a router [2]. A sequence of gateways ensuring continuous coverage can be used to synchronize remote nodes. For instance, nodes n_1 and n_2 in figure 1, which can be either regular nodes or mesh routers, do not belong to coverage area of any common reference. However, they may get synchronized through gateways g_1 and g_2. Suppose for now that nodes within a single reference's coverage area to be

[1] Refer to Eq. 1 and 2 in [10, p. 1], v_{x,λ_x}, v_{y,λ_y} would be the noise (variable delay) due to both transmission and reception. This adds transmission delays to the time critical path

[2] Note that regular nodes also act as routers in mesh networks

able to synchronize using RBS (detailed estimators for this heterogeneous environment will be provided in the next section). Nodes n_1, g_1, (respectively n_2, g_2) can directly synchronize through R_1 (respectively R_3). On the other hand, gateways, g_1, g_2, can synchronize through R_2. So it would be possible to convert time at n_1 to that at n_2. This can be generalized to larger areas with employment of several gateways. Generally speaking, it is sufficient to have a sequence of references R_1, R_2, R_{k-1}, and gateways, n_2, n_3, n_{k-1}, such that: $\forall i \in \{1, ..., k-1\}$, $n_i, n_{i+1} \in Cov_{R_i}$, where n_1 and n_k denote synchronizing nodes. In the following, a general synchronization model is provided to explain how time convergence can be achieved, assuming the previous condition holds. Consider time conversion at node n_1 to the time of node n_k; the same process is to be performed for the opposite direction. Note t_{n_i} the time of node i, read with its local clock, and $\Theta_{skew}^{n_i \to n_j}$, (respectively $\Theta_{offset}^{n_i \to n_j}$) the relative skew (respectively offset) relating node ni to nj. Time reading of nodes can therefore be related by:

$$\begin{array}{lll} t_{n_k} = & \Theta_{skew}^{n_{k-1} \to n_k} t_{n_{k-1}} + & \Theta_{offset}^{n_{k-1} \to n_k} \\ t_{n_{k-1}} = & \Theta_{skew}^{n_{k-2} \to n_{k-1}} t_{n_{k-2}} + & \Theta_{offset}^{n_{k-2} \to n_{k-1}} \\ . & . & . \\ . & . & . \\ t_{n_2} = & \Theta_{skew}^{n_1 \to n_2} t_{n_1} + & \Theta_{offset}^{n_1 \to n_1} \end{array}$$

By successive substitutions of t_{n_i} expressions in the previous equations ($i \in k-1, ..., 2$), it results,

$$t_{n_k} = \prod_{i=1}^{k-1} \Theta_{skew}^{n_i \to n_{i+1}} t_{n_1} + \sum_{i=1}^{k-2} [(\prod_{j=i}^{k-2} \Theta_{skew}^{n_{j+1} \to n_{j+2}}) \Theta_{offset}^{n_i \to n_{i+1}}] + \Theta_{offset}^{n_{k-1} \to n_k}, \quad (1)$$

Once each of ($\Theta_{skew}^{n_i \to n_j}$, $\Theta_{offset}^{n_i \to n_j}$) parameters are estimated, it would be straightforward to node n_1 to use its own reading (t_{n_1}) to estimate the corresponding time at node n_k, say t_{n_k}. Each gateway should provide the synchronizing nodes with its estimate of the appropriate parameters ($\Theta_{skew}^{n_i \to n_j}$, $\Theta_{offset}^{n_i \to n_j}$). A high level of abstraction is made to the protocol stack, and any routing protocol may be used. The only condition to be checked for route selection is that at each hop, the downstream and upstream nodes must belong to at least one common reference's coverage area. That is, to avoid reference coverage disconnection. Gateways will be those routers having different (disjoint) reference share, between their upstream and downstream links. It has been shown how to ensure global synchronization once local one is ensured. Now it remains to tackle local synchronization, and define estimation method for the relative skews and offsets between nodes belonging to the coverage area of a common reference.

IV. Skew/Offset Estimation

Estimation of relative skew and offset between two nodes, say u and v, located in the coverage area of a common reference is considered. I.e, node v's estimate of relative parameters relating its clock to that of u. The reference

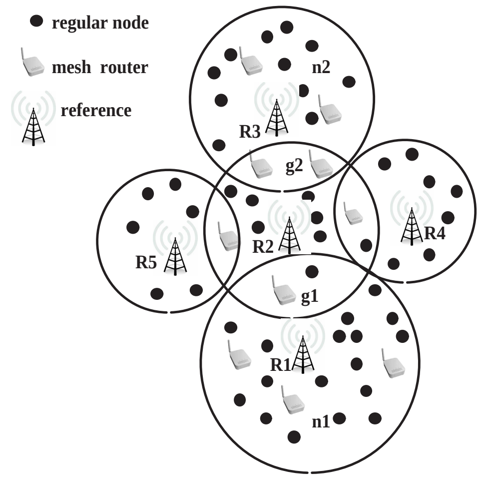

Fig. 1. General network architecture

broadcasts K signals, then the corresponding reception times are to be used by the two nodes as a sample for estimation. Let $t_{u,i}$ and $t_{v,i}$, $i = \{1..K\}$, denote the i^{th} sample's element (i^{th} signal reception time) of u and v, respectively. Each signal will inevitably experience some delay at each node before the reception event is timestamped. The delay can be decomposed to fixed portion (time necessary for decoding, memory access, etc.), and a variable portion (latency due to operating system schedule, network interface behavior, etc.) [4]. Let d_{ui} (respectively d_{vi}) denote the fixed portion of the i'th signal at node u (respectively v), and X_{ui} (respectively X_{vi}) the variable portion f the i'th signal at node u (respectively v). Times $t_{u,i}$ and $t_{v,i}$ can be related by,

$$t_{u,i} = \Theta_{skew}^{v \to u} t_{v,i} + \Theta_{offset}^{v \to u} + d_{ui} + X_{ui} - (d_{vi} + X_{vi}).$$

For simplicity, remove upper indices from the previous parameters, and denote $X_{ui} - X_{vi}$ by X_i, and $d_{ui} - d_{vi}$ by d_i. The previous equation yields,

$$X_i = t_{u,i} - \Theta_{skew} t_{v,i} - \Theta_{offset} - d_i, \quad (2)$$

Each node is supposed to know its fixed delay. Random delays are supposed to be identical distributed random variables (idrv), but with parameters that may change from a node to another (given the node heterogeneity). In this paper, Gaussian distribution is considered. Suppose $X_{ui} \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_{vi} \sim \mathcal{N}(\mu_2, \sigma_2^2)$. Consequently, $X_i \sim \mathcal{N}(\mu_1 - \mu 2, \sigma_1^2 + \sigma_2^2)$, since it is the difference of two independent and idrv following a Gaussian distribution. Nodes are supposed to separately acquire their respective parameters (μ_i, σ_i), at device equilibration phase or network initialization phase. These parameters related to the variable delay, as well as fixed delays, are exchanged between synchronizing nodes (either directly or using the appropriate routing protocol if needed). In the following, μ refers to $\mu_1 - \mu 2$ and σ^2 to $\sigma_1^2 + \sigma_2^2$.

The likelihood function, $\mathcal{L}(\Theta_{skew}, \Theta_{offset} | X_1, ...X_K)$, is,

$$\mathcal{L}(\Theta_{skew}, \Theta_{offset} | X_1, ...X_K) = \prod_{i=1}^{K} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(X_i - \mu)^2}$$

$$= (\frac{1}{\sqrt{2\pi\sigma^2}})^K (e^{\frac{-1}{2\sigma^2}\sum_{i=1}^{K}(t_{u,i} - \Theta_{skew} t_{v,i} - \Theta_{offset} - d - \mu)^2}). \quad (3)$$

Then the log-likelihood is,

$$\log \mathcal{L}(\Theta_{skew}, \Theta_{offset}|X_1, ...X_K) = K \log(\tfrac{1}{\sqrt{2\pi\sigma^2}}) - \frac{1}{2\sigma^2}\sum_{i=1}^{K}(t_{u,i} - \Theta_{skew} t_{v,i} - \Theta_{offset} - d - \mu)^2. \quad (4)$$

Since $(\widehat{\Theta}^{mle}_{skew}, \widehat{\Theta}^{mle}_{offset}) =$

$$argmax(\log \mathcal{L}(\Theta_{skew}, \Theta_{offset}|X_1, ...X_K)), \quad (5)$$

the estimators may be obtained by differentiating the log-likelihood function with respect to each parameter and vanishing each derivative. I.e., resolving the system of equations,

$$\tfrac{\partial \ln \mathcal{L}(\Theta_{skew}, \Theta_{offset}|X_1, ...X_K)}{\partial \Theta_{skew}} = 0,$$

$$\tfrac{\partial \ln \mathcal{L}(\Theta_{skew}, \Theta_{offset}|X_1, ...X_K)}{\partial \Theta_{offset}} = 0.$$

The resolution of the previous system yields,

$$\widehat{\Theta}^{mle}_{skew} = \frac{K\sum_{i=1}^{K} t_{v,i}(t_{u,i} - d - \mu) - \sum_{i=1}^{K} t_{v,i} \sum_{i=1}^{K}(t_{u,i} - d - \mu)}{K\sum_{i=1}^{K} t^2_{v,i} - (\sum_{i=1}^{K} t_{v,i})^2}, \quad (6)$$

$$\widehat{\Theta}^{mle}_{offset} = \frac{\sum_{i=1}^{K} t_{u,i} - \widehat{\Theta}^{mle}_{skew} t_{v,i} - d - \mu}{K}. \quad (7)$$

V. Numerical Analysis

A. Cramer-Rao Lower-bounds (CRLB)

The CRLB has been chosen to compare with as it represents the lower bound of mean square errors (MSE), or variance, for all unbiased estimators. In the following, the CRLB for both skew and offset estimators will be calculated. It will be used later for comparison as a theoretical optimum. The CRLB of the vector, $[\Theta_{skew}, \Theta_{offset}]^T$, denoted in the following by $[\Theta_1, \Theta_2]^T$, can be calculated from the inverse of the 2×2 Fisher information vector, I^{-1}. I is defined by,

$$I_{i,j} = E[\tfrac{\partial \ln \mathcal{L}(x,\Theta_1,\Theta_2)}{\partial \Theta_i} \tfrac{\partial \ln \mathcal{L}(x,\Theta_1,\Theta_2)}{\partial \Theta_j}] = -E[(\frac{\partial^2 \ln \mathcal{L}(x, \Theta_1, \Theta_2)}{\partial \Theta_i \partial \Theta_j})]. \quad (8)$$

Derivatives may be calculated from Eq. 4, to obtain I^{-1}. The bounds of the estimators are,

$$Var(\widehat{\Theta}_{skew}) \geq (I^{-1})_{1,1}$$

$$(I^{-1})_{1,1} = \frac{K\sigma^2}{K\sum_{i=1}^{K} t^2_{v,i} - (\sum_{i=1}^{K} t_{v,i})^2} \quad (9)$$

$$Var(\widehat{\Theta}_{offset}) \geq (I^{-1})_{2,2}$$

$$(I^{-1})_{2,2} = \frac{\sigma^2 \sum_{i=1}^{K} t^2_{v,i}}{K\sum_{i=1}^{K} t^2_{v,i} - (\sum_{i=1}^{K} t_{v,i})^2} \quad (10)$$

B. Simulation

To evaluate the proposed estimators and compare them with the theoretical bound (CRLB), a MATLAB simulation has been carried out. The mean square error (MSE) of the proposed estimators (MLE) and CRLB have been measured. The error in MSE refers to the difference between real value and the estimated one, and depicts the estimator's precision. First, the number of signals (sample's size) used for the estimation has been varied to measure the MSE, while μ_1 and μ_2 have been fixed to 1 and 2 ms (respectively), i.e. μ fixed to 1 ms. Then the number of signals have been fixed to its average value (50), and μ have been varied from 0 to 100ms. Each point of the plots presented herein is the average of 10^5 measurements. Figures 2, 3 show that both estimators quadratically converge to the CRLB, and to zero, as the number of signals (signals) increases. They converge very fast, notably the skew estimator (Figure 2) whose difference from the CRLB is almost invisible even with the logarithmic scale. Figures 4 and 5 show that the MSEs are unaffected by the increase of μ, and thus such is the estimators' precision. They also confirm low difference from the CRLB, which is at the order of 10^{-2} for the offset and 10^{-5} for the skew.

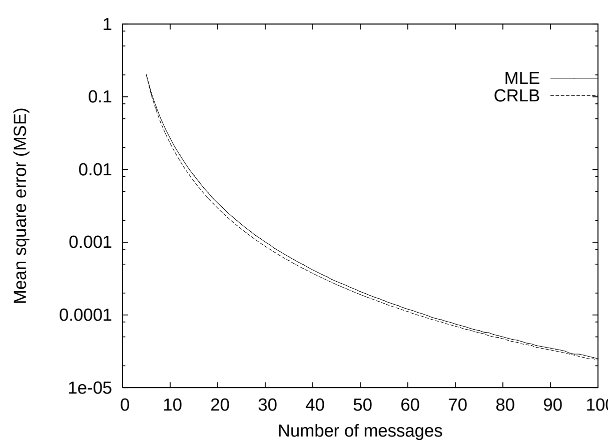

Fig. 2. MSE of $\widehat{\Theta}_{skew}$ vs. number of signals

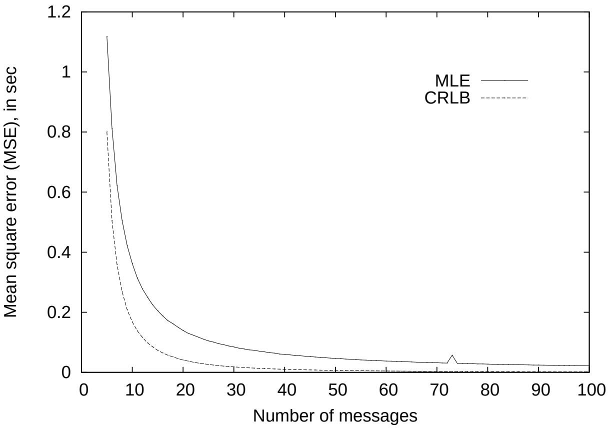

Fig. 3. MSE of $\widehat{\Theta}_{offset}$ vs. number of signals

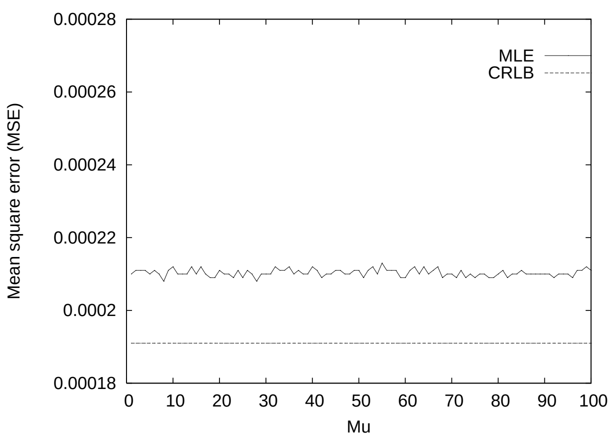

Fig. 4. MSE of $\widehat{\Theta}_{skew}$ vs. μ

VI. Conclusion

The use of reference broadcast synchronization (RBS) in general heterogeneous wireless networks has been considered in this paper. RBS ensures high precision by eliminating the sender's uncertainty from the critical path. A model for mesh networks has been defined, which extends to any wireless multi-hop heterogeneous network. The model generalizes RBS to multi-hop synchronization, and considers delay variation between nodes caused by the heterogeneity of the nodes and the environment. In contrast to state-of-the-art RBS estimators for the joint skew/offset model, the proposed ones enable to directly estimate relative parameters between nodes without turning to synchronizing to the reference. This accurately translates RBS principle (receiver/receiver synchronization paradigms). Using the proposed model, maximum likelihood estimators (MLEs) for the skew and offset have been calculated, as well as the corresponding Cramer-Rao lower bounds (CRLB). The later is the theoretical bound for the precession of any unbiased estimator. The estimators have been numerically compared to their respective CRLB. Results demonstrate fast and quadratic convergence of the estimators as the sample size (number of signals used to calculate the estimate) increases. They also show that the precision is unaffected by the increase of reception delay difference, which reflects the degree of heterogeneity. This confirm the suitability of the estimators to heterogeneous environments. Evaluating the precision of the estimators in multi-hop scenario is a perspective to this work. Considering other delay distribution- such as exponential delays- is also a potential perspective. Finally, note that the MLE when the delays' means are unknown does not exist, as the likelihood function does not have a unique maximum. Defining other estimators- that may be biased- is another perspective.

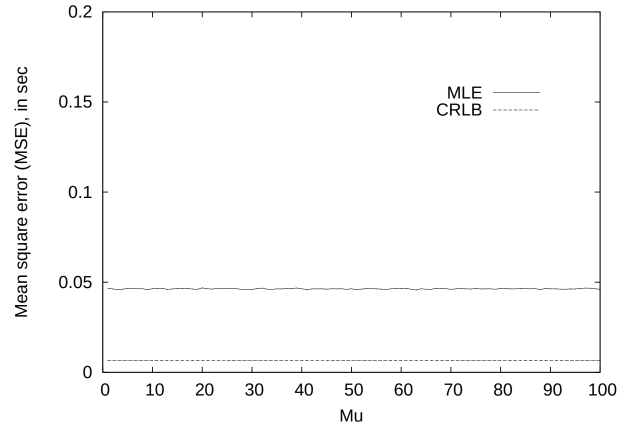

Fig. 5. MSE of $\widehat{\Theta}_{offset}$ vs. μ

References

[1] C. Lenzen, T. Locher, P. Sommer, and R. Wattenhofer, "Clock synchronization: Open problems in theory and practice," in *Proceedings of the 36th Conference on Current Trends in Theory and Practice of Computer Science (SOFSEM '10)*, ser. LNCS 5901. Berlin, Heidelberg: Springer-Verlag, 2010, pp. 61–70.

[2] O. Simeone, U. Spagnolini, Y. Bar-Ness, and S. H. Strogatz, "Distributed synchronization in wireless networks," *IEEE Signal Processing Magazine*, vol. 25, no. 5, pp. 81–97, 2008.

[3] J. Elson, L. Girod, and D. Estrin, "Fine-grained network time synchronization using reference broadcasts," in *5th USENIX Symposium on Operating System Design and Implementation (OSDI'02)*, December 2002.

[4] B. Sundararaman, U. Buy, and A. D. Kshemkalyani, "Clock synchronization for wireless sensor networks: a survey," *Ad hoc Networks*, vol. 3, no. 3, pp. 281–323, 2005.

[5] J. Ridoux and D. Veitch, "Principles of robust timing over the internet," *Communications of the ACM*, vol. 53, pp. 54–61, May 2010.

[6] D. Jeske, "On maximum-likelihood estimation of clock offset," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 53–54, 2005.

[7] K.-L. Noh, Q. M. Chaudhari, E. Serpedin, and B. W. Suter, "Novel clock phase offset and skew estimation using two-way timing message exchanges for wireless sensor networks," *IEEE Transactions on Communications*, vol. 55, no. 4, pp. 766–777, 2007.

[8] M. Leng and Y.-C. Wu, "On clock synchronization algorithms for wireless sensor networks under unknown delay," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 1, pp. 182–190, 2010.

[9] S. Ganeriwal, I. Tsigkogiannis, H. Shim, V. Tsiatsis, M. B. Srivastava, and D. Ganesan, "Estimating clock uncertainty for efficient duty-cycling in sensor networks," *IEEE/ACM Transations on Networking.*, vol. 17, pp. 843–856, June 2009.

[10] I. Sari, E. Serpedin, K.-L. Noh, Q. M. Chaudhari, and B. W. Suter, "On the joint synchronization of clock offset and skew in rbs-protocol," *IEEE Transactions on Communications*, vol. 56, no. 5, pp. 700–703, 2008.